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Title: Applications of Analytic Models to Spent Fuel Cask Analysis

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Applications of Analytic Models to Spent Fuel Cask Analysis

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NNSA National Nuclear Security Administration

Managed by Triad National Security, LLC for the U.S. Department of Energy's NNSA

Biography

- 2014 B.S. Engineering Physics from Colorado School of Mines
  - 2 papers
  - 1 patent
- 2017 M.S. Nuclear Engineering from University of Florida
  - 2 Internships at LANL
    - Plasma Physics
      - “Pulse Dilation Technique on Gas Cherenkov Detectors for application in Inertial Confinement Fusion”
      - Award of Recognition
    - Theoretical Physics
      - “Determining the Validity of Diffusion Approximated Flux Values”
  - Poster Presentation at ANS Winter Conference 2016
  - Presentation at ANS Student Conference 2018
  - Currently Working with LANL for my PhD

Proposal Agenda

- Introduction
  - Motivation
  - Project Goals
- Background
  - HI-STORM 100
  - Neutron Transport
  - Symmetry Analysis
  - Sensitivity Analysis
- Current Work
  - MCNP
  - Analytic
- Future Work

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Introduction

Introduction

- Explores the novel intersection of three areas of physics and mathematics
  - Neutron transport theory
  - Symmetry analysis techniques (Lie Group)
  - Sensitivity analysis
- Motivation
  - Simulation has become a powerful tool for analyzing complex systems
  - These tools model continuous calculus using algebraic equations
    - Introduces assumptions and approximations into a simulation program
    - Need to ensure simulations do not violate assumptions and approximations
  - Users are also capable of making errors
    - Simplifications in modeling
    - Developing input files
  - Comparison with experimental data is the best way to ensure simulations were conducted correctly
  - Sometimes experimental data is difficult to obtain if any exists at all
    - Such as with spent fuel casks

Introduction

- When no experimental data is available
  - We can use analytic or semi-analytic models to compare simulations against
  - If the analytic results agree with simulation results
    - Confidence in gained in simulation results and in model input
    - The user understands the physics that is occurring
    - User is capable of analyzing simulation results appropriately
  - If the two disagree
    - The user learns:
      - Important physics was overlooked
      - Input may be wrong
    - The user learns about the problem
    - Leads to a deeper understanding and more in-depth analysis

Introduction

- Comparisons between computational and analytic results are further exemplified through a sensitivity analysis
- Computational sensitivity analysis
  - Requires identifying possible parameters that could affect the results
  - Developing a new model for each parameter variation
  - Result, is resource intensive
- Analytic sensitivity analysis
  - Requires identifying possible parameters that could affect the results
  - By using a generalized form of a directional derivative, parameter sensitivities can be calculated directly
  - Result, is less intensive than for computational sensitivity analysis
- When the results of sub-region and sensitivity analysis compare favorably, confidence is gained in computational analysis

Introduction

- The aforementioned processes are generic and can be applied to any system governed by differential equations
- As a proof of principal, these processes will be applied to a Holtec HI-STORM 100 spent fuel cask
- The goal of current work is to seek a sub-region of a detailed problem
  - Simplifications are applied to the Boltzmann transport equation (BTE) for neutrons
  - Solutions are compared to computational solutions of the sub-region
  - Captures elemental physical processes occurring in the sub-region of the full-scale problem

https://holtecinternational.com/productsandservices/wasteandfuelmanagement/dry-cask-and-storage-transport/hi-storm/hi-storm-100/

Introduction: Project Goals

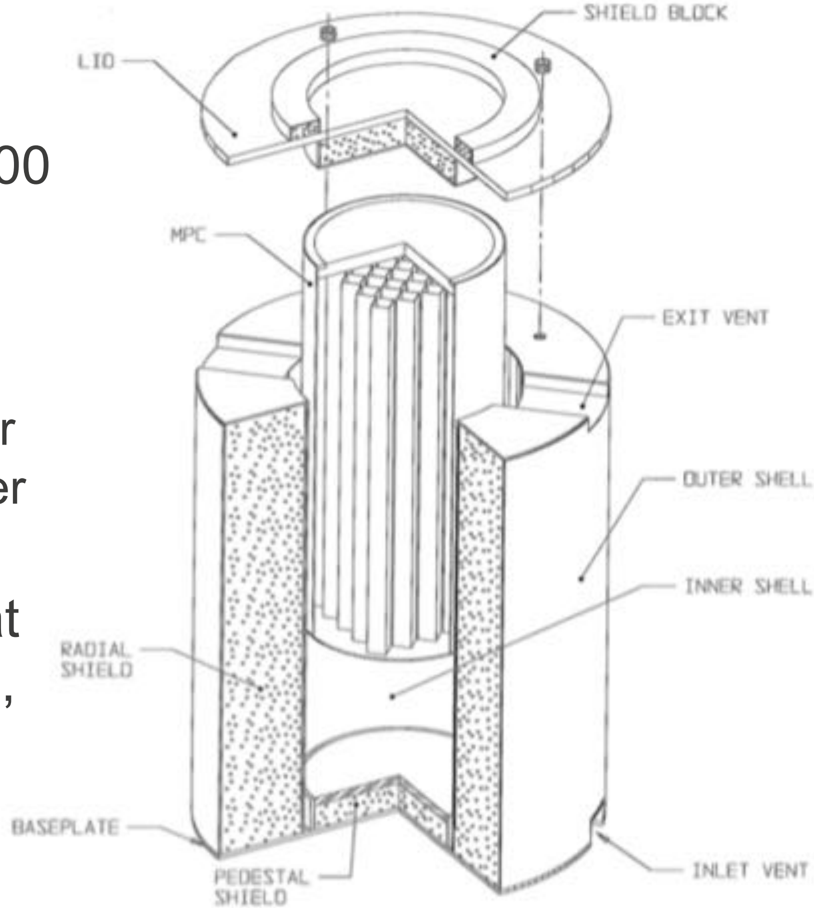
- To develop a methodology for using analytic models to verify simulation results
  1. Identification of sub-regions where BTE can be applied
  2. Solutions to BTE
     - Symmetry analysis procedure
  3. Comparison of computation and analytic results
  4. Sensitivity analysis of simulation models
  5. Sensitivity analysis of analytic models
  6. Comparison of simulation and analytic sensitivity analysis results
  7. Analysis of underlying physics of HI-STORM 100 spent fuel cask

- The proposed methodology develops a novel procedure for analyzing simulation results when no experimental data can be acquired

Background

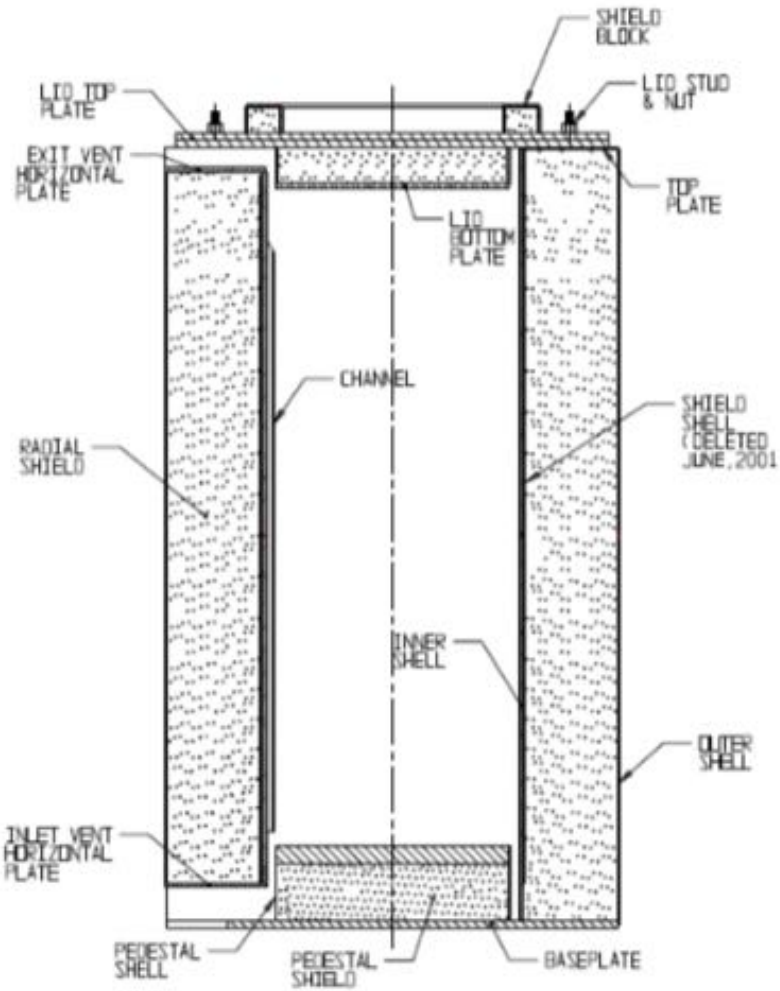
Background: HI-STORM 100

- Simulation has become prevalent in spent fuel cask analysis
  - Radiation shielding capabilities
  - Fuel shifting
  - Imaging the interior of a cask
- Holtec International HI-STORM 100 spent fuel cask was chosen
  - Most used spent fuel cask storage system
  - Used to store fuel from boiling water reactors (BWR) or pressurized water reactors (PWR)
- Provides radiation protection, heat transfer, environmental protection, fuel security, and accident protection (i.e. if the cask were dropped)



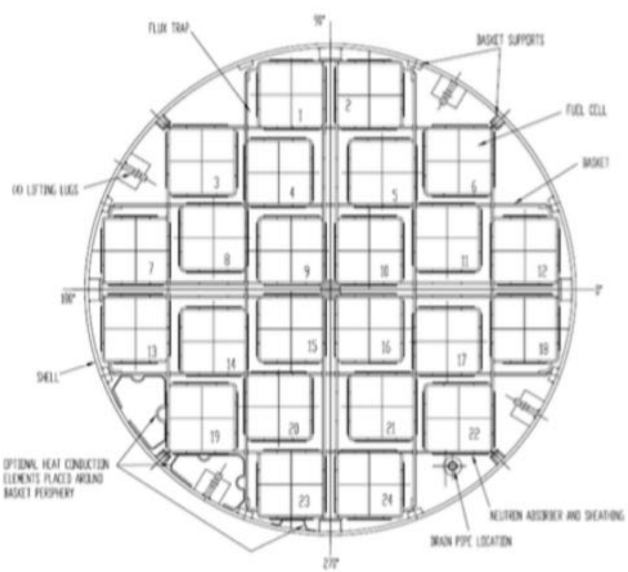
Background: HI-STORM 100

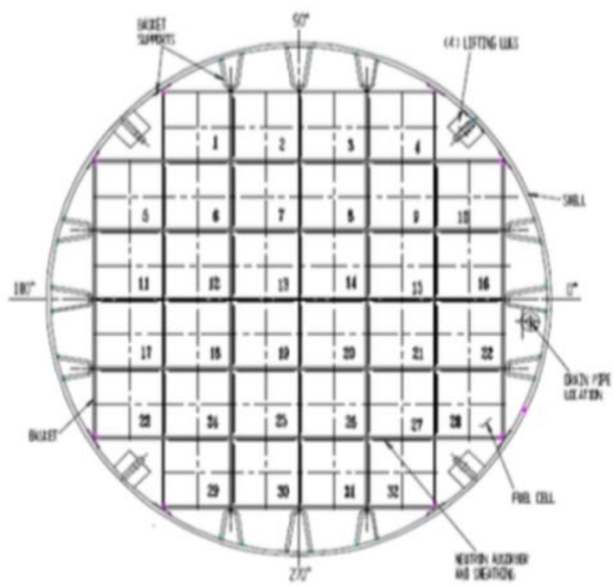
- The overpack:
  - Inner anulus of concrete
  - Outer shell of carbon steel
  - Neutron and gamma shielding above and below fuel region
  - Inner channels for air to flow through
  - Multi-purpose canister (MPC) holds spent fuel in center of overpack
- Concrete provides:
  - Neutron shielding
  - Protection in the event the cask is dropped
- Steel provides:
  - Gamma shielding
  - Structure to the cask
  - Protection to overpack and MPC

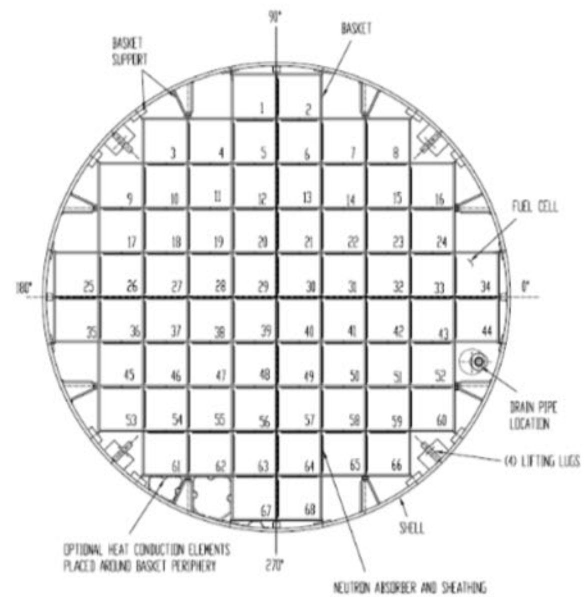


Background: HI-STORM 100

- Three main types of MPC
  - MPC-24: Used for PWR fuel
  - MPC-32: Used for PWR fuel (chosen for proposed work)
  - MPC-68: Used for BWR fuel
- Honey-comb, stainless steel structure supports fuel and provides heat transfer
- Boral pad (neutron absorber) placed between cells







Background: Neutron Transport

- The focus of current work is to identify a sub-region where analytic models can be used to verify simulation results
  - The proposed work is focused on neutron transport
  - Analytic models are based on the BTE
- Using a heuristic approach to derive BTE in phase-space defined by dV, dE, d$\hat{\boldsymbol{\Omega}}$, and dt
- BTE is a balance equation
  1. Gain Mechanics
     a) All neutron sources in dV
     b) Neutrons streaming into dV through an infinitesimal surface dS
     c) Neutrons in a different phase space entering dV, dE, d$\hat{\boldsymbol{\Omega}}$, dt
  2. Loss Mechanics
     a) Neutrons leaking out of dV through dS
     b) Neutrons undergoing an interaction in dV

Background: Neutron Transport

- Source term
- $(a) = \left[\int_V d^3r\, s(\boldsymbol{r}, E, \widehat{\boldsymbol{\Omega}})\right] dE d\widehat{\boldsymbol{\Omega}}$

- Interaction term
- $(e) = \left[\int_V d^3r\, \Sigma_t(\boldsymbol{r}, E)\varphi(\boldsymbol{r}, E, \widehat{\boldsymbol{\Omega}})\right] dE d\widehat{\boldsymbol{\Omega}}$

- In-scattering term
- $(c) = \left[\int_V d^3r \int_{4\pi} d\widehat{\boldsymbol{\Omega}} \int_0^\infty dE'\, \Sigma_s(E' \to E, \widehat{\boldsymbol{\Omega}}' \to \widehat{\boldsymbol{\Omega}})\varphi(\boldsymbol{r}, E', \widehat{\boldsymbol{\Omega}}')\right] dE d\widehat{\boldsymbol{\Omega}}$

- Streaming term
- $(d) - (b) = \left[\int_V d^3r \widehat{\boldsymbol{\Omega}} \cdot \varphi(\boldsymbol{r}, E, \widehat{\boldsymbol{\Omega}})\right] dE d\widehat{\boldsymbol{\Omega}}$

$$(a) + (b) + (c) - (d) - (e) = 0$$

Background: Neutron Transport

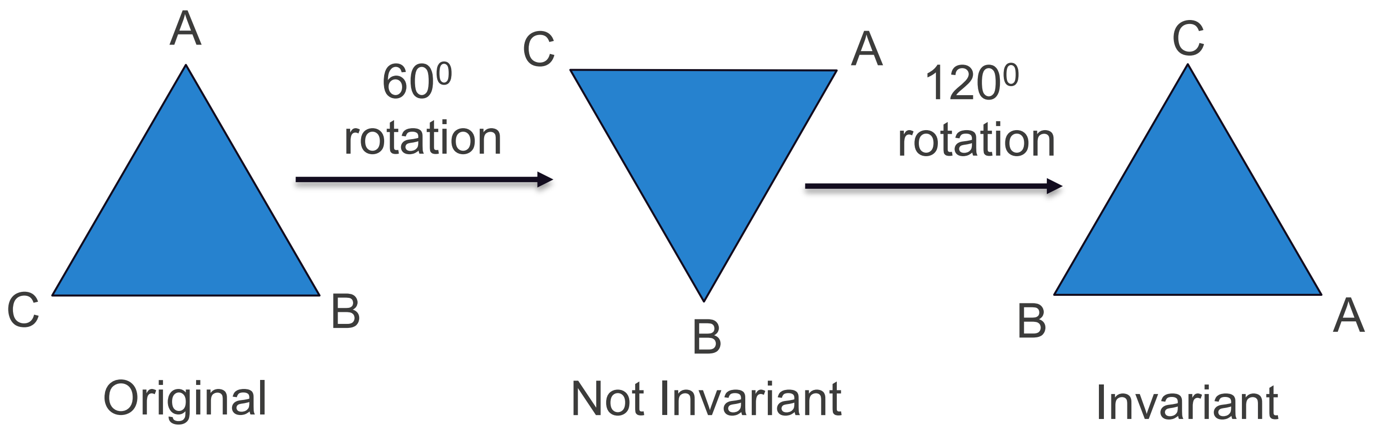
- The steady-state BTE for neutrons is:

$$\widehat{\boldsymbol{\Omega}} \cdot \boldsymbol{\nabla}\varphi(r, E, \widehat{\boldsymbol{\Omega}}) + \Sigma_t \varphi(r, E, \widehat{\boldsymbol{\Omega}})$$
$$= \int_{4\pi} d\widehat{\boldsymbol{\Omega}} \int_0^\infty dE'^{\Sigma_s}\left(E' \to E, \widehat{\boldsymbol{\Omega}}' \to \widehat{\boldsymbol{\Omega}}\right)\varphi(r, E', \widehat{\boldsymbol{\Omega}}') + s(r, E, \widehat{\boldsymbol{\Omega}})$$

- First-order, linear, integro-differential equation
  - One of the most difficult types of problems to solve directly
- Applying assumptions allow us to solve the BTE
  - Any assumption we apply will not hold across the full problem
- Identifying a sub-region allows us to apply assumptions
  - In an appropriate sub-region, assumptions will hold
  - The BTE can be reduced to a tractable form
- If assumptions can be relaxed, we can use a more accurate form of the BTE
  - Symmetry analysis becomes useful

Background: Symmetry Analysis

- Many partial-differential equation solving techniques rely on manipulating an equation into a form for which a solution is known
  - Becomes difficult or impossible as equations become more complex
- Symmetry analysis provides a more standardized approach
  - Change of variables
    - The equation is mapped into a new coordinate system
  - Invariance
    - When an equation is unchanged under the action of an operation
    - Solutions to new equation will be solutions to old equation



Background: Symmetry Analysis

- Invariance Example

$$F\left(x, y, \frac{\partial y}{\partial x}\right) = \frac{\partial y}{\partial x} - e^{x-y} = 0$$

- Transformation operations:

$$x = \tilde{x} + s, \qquad y = \tilde{y} + s, \qquad \frac{\partial y}{\partial x} = \frac{\partial \tilde{y}}{\partial \tilde{x}}$$

- Applying transformation operations

$$F\left(x, y, \frac{\partial y}{\partial x}\right) = \frac{\partial \tilde{y}}{\partial \tilde{x}} - e^{(\tilde{x}+s)-(\tilde{y}+s)} = \frac{d\tilde{y}}{d\tilde{x}} - e^{\tilde{x}-\tilde{y}} = \tilde{F}\left(\tilde{x}, \tilde{y}, \frac{\partial \tilde{y}}{\partial \tilde{x}}\right)$$

- The two functions are the same
  - An example of a translation symmetry

Background: Symmetry Analysis

- We are looking for symmetries which leave our equation invariant
- These symmetries can be found systematically
- Introducing an example:

$$F\left(x, y, \frac{dy}{dx}\right) = \frac{dy}{dx} - \frac{y}{x} - \tan\left(\frac{y}{x}\right) = 0$$

- For simplicity, we define $z \coloneqq \frac{dy}{dx}$ and the transformations

$$\tilde{x} \equiv \alpha(x, y, z;\ \varepsilon), \tilde{y} \equiv \beta(x, y, z;\ \varepsilon), \tilde{z} \equiv \gamma(x, y, z;\ \varepsilon)$$

- Determining general transformations is difficult if not impossible!
- Sophus Lie discovered the localized evaluation is equivalent to finding α, β, and γ
  - Through use of Taylor expansion

https://en.wikipedia.org/wiki/Sophus_Lie

Background: Symmetry Analysis

- Taking the Taylor expansion about $\varepsilon = 0$

$$\tilde{F} = F + \epsilon \frac{\partial \tilde{F}}{\partial \epsilon}\bigg|_{\epsilon=0} + \frac{\epsilon^2}{2} \frac{\partial^2 \tilde{F}}{\partial \epsilon^2}\bigg|_{\epsilon=0} + \mathcal{O}(\epsilon^3)$$

- Evaluating the derivatives

$$\tilde{F} - F = \varepsilon \left[\eta \frac{\partial}{\partial x} + \phi \frac{\partial}{\partial y} + \zeta \frac{\partial}{\partial z}\right] F + \varepsilon^2 \left[\eta \frac{\partial}{\partial x} + \phi \frac{\partial}{\partial y} + \zeta \frac{\partial}{\partial z}\right]^2 F + \mathcal{O}(\varepsilon^3);$$

$$\eta = \frac{\partial \alpha}{\partial \varepsilon}, \phi = \frac{\partial \beta}{\partial \varepsilon}, \zeta = \frac{\partial \gamma}{\partial \varepsilon}$$

- We define the prolonged group generator

$$pr\boldsymbol{X} \equiv \eta \frac{\partial}{\partial x} + \phi \frac{\partial}{\partial y} + \zeta \frac{\partial}{\partial z}$$

- An invariant operation on F means $\tilde{F} - F = 0$

Background: Symmetry Analysis

- To solve $pr\boldsymbol{X}\{\boldsymbol{F}\} = 0$, we return to our example

$$pr\boldsymbol{X}\{F\} = \eta\left[\frac{y}{x^2} + \frac{y}{x^2}sec^2\left(\frac{y}{x}\right)\right] + \phi\left[\frac{1}{x} + \frac{1}{x}sec^2\left(\frac{y}{x}\right)\right] + \zeta = 0$$

- A solution for η, ϕ, and ζ is

$$\eta = x, \qquad \phi = y, \qquad \zeta = 0$$

- The group generator is then

$$\boldsymbol{X} = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial x}$$

- To construct our similarity variable, we apply $\boldsymbol{X}$ to some function, $F(x, y)$ and set $\boldsymbol{X}\{F\} = 0$
- The previous step ensures symmetries found in F, will be the same as the symmetries in our problem

Background: Symmetry Analysis

$$\boldsymbol{X}\{F\} = x\frac{\partial F}{\partial x} + y\frac{\partial F}{\partial y} = 0$$

- Rearranging terms produces our characteristic system

$$\frac{\partial F}{0} = \frac{\partial x}{x} = \frac{\partial y}{y}$$

- Solving the characteristic system will yield constants
  - Function of independent and dependent variables
- These constants are called similarity variables
  - Used to simply our original problem
- Solving

$$\frac{\partial F}{0} = \frac{\partial x}{x}, \qquad \frac{\partial x}{x} = \frac{\partial y}{y}$$

yield

$$F = constant, \qquad r = \frac{y}{x}$$

Background: Symmetry Analysis

- The derivative can be re-written as

$$\frac{\partial y}{\partial x} = \frac{\partial r}{\partial x}x + r$$

- Re-writing the original equation in terms of r and x

$$\frac{\partial r}{\partial x}x - \tan(r) = 0$$

- This is a separable equation with the solution

$$r = sin^{-1}(cx);\ c \equiv constant$$

- Re-writing the solution in the original co-ordinate system

$$y = xsin^{-1}(cx)$$

- We now arrive at the solution to the original equation

Background: Symmetry Analysis

- Procedure:

1. Re-write $F\left(x, y, \frac{\partial y}{\partial x}\right)$ as $F(x, y, z)$
2. Apply the prolonged group generator to set up determining equations
3. Solve the determining equations for η, ϕ and ζ
4. Apply the completed group generator to find the determining system
5. Solve the characteristic system to find similarity variables
6. Re-express F in terms of the similarity variables and arrive at a simplified expression
7. Solve the simplified expression
8. Re-express the solution to the simplified expression in the original co-ordinate system

Background: Sensitivity Analysis

- A physical system is modeled by linear or non-linear differential equations
  - Define the system's response to an input based on parameters
- Typically, there is a level of uncertainty attached to each parameter
- The practice of ascertaining the behavior of a system in response to parameter variations is known as sensitivity analysis
- In this work, we use a procedure developed by Dan Cacuci
  - Based on a direct correspondent between local sensitivity analysis and Gâteaux-derivative (G-derivative)
  - Cacuci's method is more general and less computationally intensive than other methods
- Sensitivities can be used to
  - Rank parameters by importance
  - Asses the change in response due to parameter variation
  - Perform uncertainty analysis

Background: Sensitivity Analysis

- G-derivative

$$\delta R(x_0; h) \equiv \lim_{t \to 0} \frac{R(x_0 + th) - R(x_0)}{t}$$

- The G-derivative is a generalization of a directional derivative
- Only need the first G-derivative of our functions to find sensitivities
- Cacuci developed a procedure called the Forward Sensitivity Analysis Procedure (FSAP)
  - Solving the Forward Sensitivity Equations (FSE) determines the system's response to a single variation
  - Needs to be repeated for different variations of parameters
  - Repeated solving of FSE for each parameter variation constitutes the FSAP

Background: FSAP

- A system is described by coupled operator equations

$$\boldsymbol{L}(\boldsymbol{\alpha})\boldsymbol{u} = \boldsymbol{Q}[\boldsymbol{\alpha}(x)]$$

- Boundary conditions are used to solve the previous system of equations

$$\boldsymbol{B}(\boldsymbol{\alpha})\boldsymbol{u} - \boldsymbol{A}(\boldsymbol{\alpha}) = 0$$

- Taking the first G-derivative yields

$$\boldsymbol{L}(\boldsymbol{\alpha}^0)\boldsymbol{h}_u + [\boldsymbol{L}'_\alpha(\boldsymbol{\alpha}^0)\boldsymbol{u}^0]\boldsymbol{h}_\alpha - \delta\boldsymbol{Q}(\boldsymbol{\alpha}^0; \boldsymbol{h}_\alpha) = \boldsymbol{0}$$
$$\boldsymbol{B}(\boldsymbol{\alpha}^0)\boldsymbol{h}_u + [\boldsymbol{B}'(\boldsymbol{\alpha}^0)\boldsymbol{u}^0]\boldsymbol{h}_\alpha - \delta\boldsymbol{A}(\boldsymbol{\alpha}^0; \boldsymbol{h}_\alpha\,) = \boldsymbol{0}$$

- Solving for $\boldsymbol{h}_u$ allows us to find the sensitivities

$$\delta\boldsymbol{R}(\boldsymbol{h}) \equiv \boldsymbol{R}'_\alpha\boldsymbol{h}_\alpha + \boldsymbol{R}'_u\boldsymbol{h}_u$$

Background: FSAP Example

- Consider the following:

$$D\frac{d^2\varphi}{dx^2} - \Sigma_a\varphi + S = 0; x \in (-a, a)$$

- with the boundary condition

$$\varphi(\pm a) = 0$$

- A detector placed within the slab would read

$$R(\varphi, \boldsymbol{\alpha}) \equiv \Sigma_d\varphi(b); 0 < b < |a|$$

- The parameters are:

$$\boldsymbol{\alpha} \equiv (\Sigma_a, D, S, \Sigma_d)$$

Background: FSAP Example

- The nominal flux is found from solving the diffusion approximation

$$\varphi^0(x) = \frac{S^0}{\Sigma_a^0}\left(1 - \frac{\cosh\left(b\sqrt{\frac{\Sigma_a^0}{D^0}}\right)}{\cosh\left(a\sqrt{\frac{\Sigma_a^0}{D^0}}\right)}\right)$$

- The nominal response is then

$$R^0(\varphi^0, \boldsymbol{\alpha}^0) = \Sigma_d^0 \varphi^0(x = b)$$

- We define the variation of the parameters to be

$$\boldsymbol{h}_\alpha \equiv (\delta\Sigma_a, \delta D, \delta S, \delta\Sigma_d)$$

Background: FSAP Example

- We apply the G-derivative to the response

$$\delta R(\varphi^0, \boldsymbol{\alpha}^0; \boldsymbol{h}) = \frac{d}{dt} R\big((\varphi^0, \boldsymbol{\alpha}^0) + t\boldsymbol{h}\big); h \equiv \big(h_\varphi, h_\alpha\big)$$

- Evaluating yields

$$\delta R(\varphi^0, \boldsymbol{\alpha}^0; \boldsymbol{h}) = \boldsymbol{R'}_\alpha(\varphi^0, \boldsymbol{\alpha}^0)\boldsymbol{h}_\alpha + \boldsymbol{R'}_\varphi(\varphi^0, \boldsymbol{\alpha}^0)\boldsymbol{h}_\varphi$$

- The first term on the RHS is the “direct-effect” term

$$\boldsymbol{R'}_\alpha(\varphi^0, \boldsymbol{\alpha}^0)\boldsymbol{h}_\alpha = \delta\Sigma_d \varphi^0(x = b)$$

- The second term is the “indirect-effect” term

$$\boldsymbol{R'}_\varphi(\varphi^0, \boldsymbol{\alpha}^0)\boldsymbol{h}_\varphi = \Sigma_d^0 h_\varphi(x = b)$$

- The direct-effect term can be calculated
- h_φ needs to be found

Background: FSAP Example

- Use the definitions of the FSE

$$L(\boldsymbol{\alpha^0})h_\varphi + [L'_\alpha(\boldsymbol{\alpha}^0)\varphi^0]\boldsymbol{h}_\alpha = \mathcal{O}(\boldsymbol{h}_\alpha)^2;\ L(\boldsymbol{\alpha}^0) \equiv D^0 \frac{d^2}{dx^2} - \Sigma_a^0$$

- With the boundary condition

$$h_\varphi(\pm\alpha) = 0$$

- The second term on the LHS is

$$[L'_\alpha(\boldsymbol{\alpha}^0)\varphi^0]\boldsymbol{h}_\alpha \equiv \delta D \frac{d^2\varphi^0}{dx^2} - \delta\Sigma_a\varphi^0 + \delta S$$

- Solving the boundary value problem for h_φ

$$h_\varphi(x) = C_1(\cosh(xk) - \cosh(ak)) + C_2(x\sinh(xk)\cosh(ak) - a\sinh(ak)\cosh(xk));$$

$$C_1 = \frac{\left(\frac{\delta\Sigma_a S^0}{\Sigma_a^0} - \delta S\right)}{\Sigma_a^0(\cosh(ak))}, C_2 = \frac{\left(\frac{\delta D}{D^0} - \frac{\delta\Sigma_a}{\Sigma_a^0}\right)S^0}{2\sqrt{D^0\Sigma_a^0}(\cosh(ak))^2}, k = \sqrt{\frac{\Sigma_a^0}{D^0}}$$

Background: FSAP Example

- We now can write the expression for the sensitivities

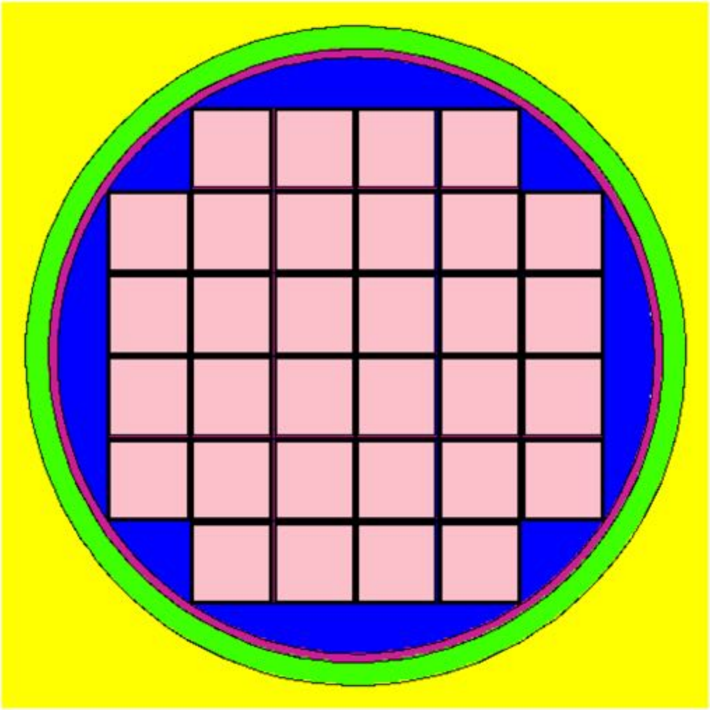
$$\delta R\big((\varphi^0, \boldsymbol{\alpha}^0); \boldsymbol{h}\big) = \delta\Sigma_d \varphi^0(x = b) + \Sigma_d^0 h_\varphi(x = b)$$

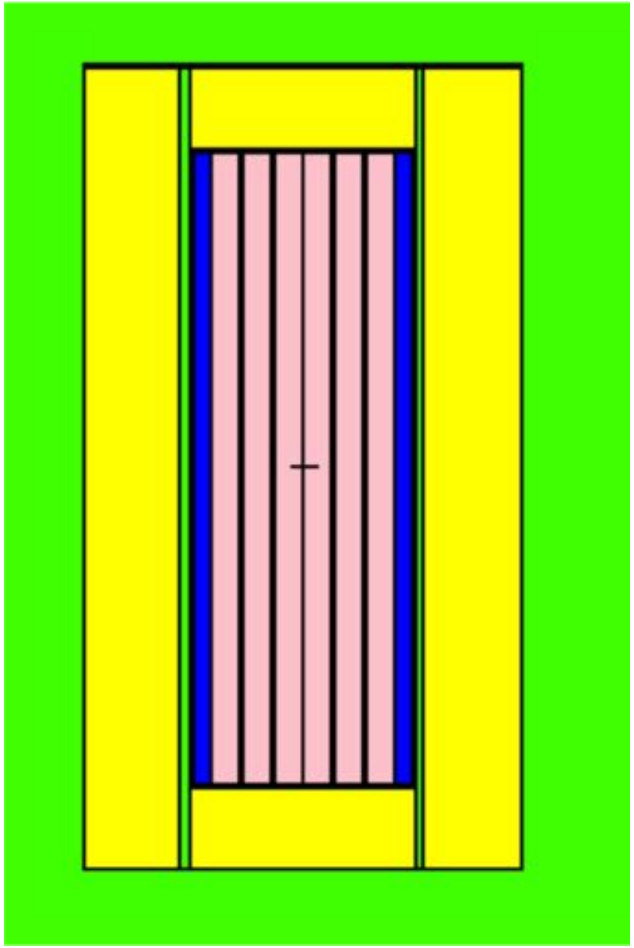
- Repeat this process for all variations of parameters in $\boldsymbol{h}_\varphi$
- Knowing the importance of each parameter can
  - Guide future model development to decrease uncertainty in the most important parameters
  - Possibly reduce analytic models to include only the most important parameters
    - Reduce computational resources needed to evaluate a problem
- Results from the FSAP will be compared to computational sensitivity analysis results
  - In computational work, various simulation inputs will be made
    - Parameters will be varied to investigate the effects on simulation results

Current Work

Current Work: MCNP

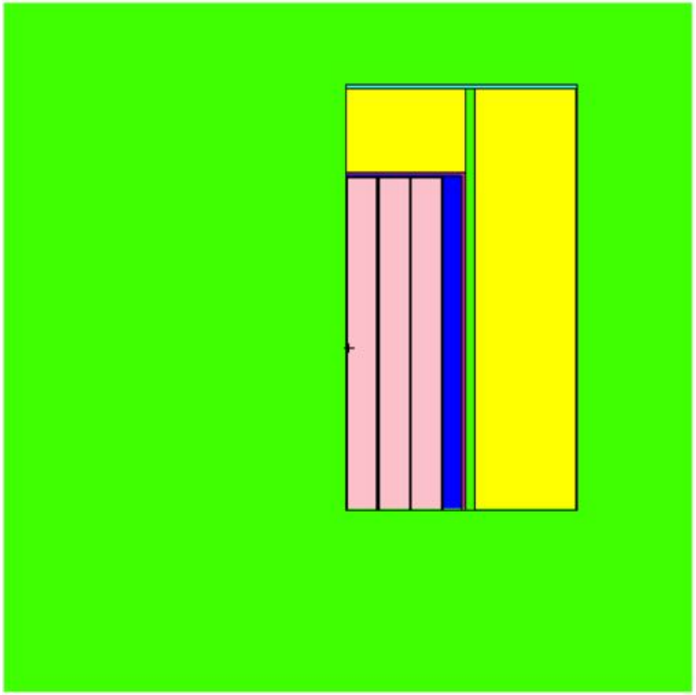
- The HI-STORM 100 spent fuel cask was simulated in MCNP
- Geometry was simplified for simulations

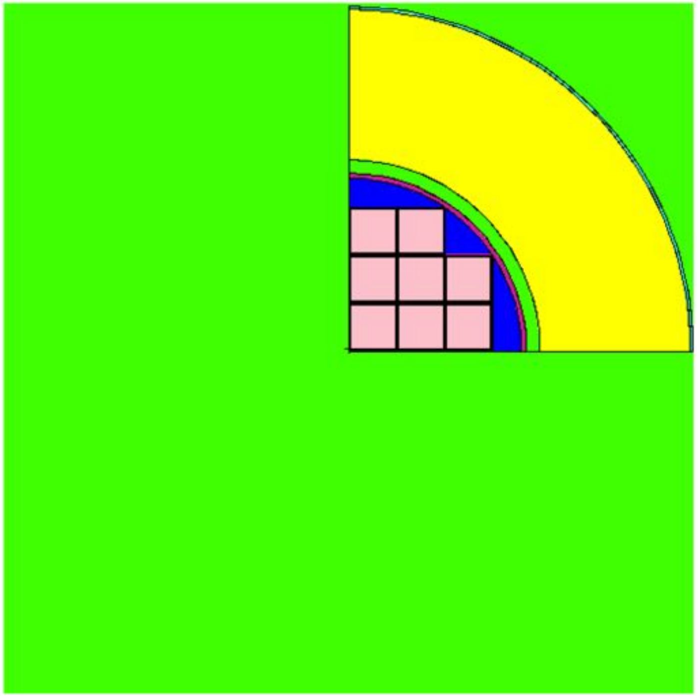




Current Work: Simulation Reduction

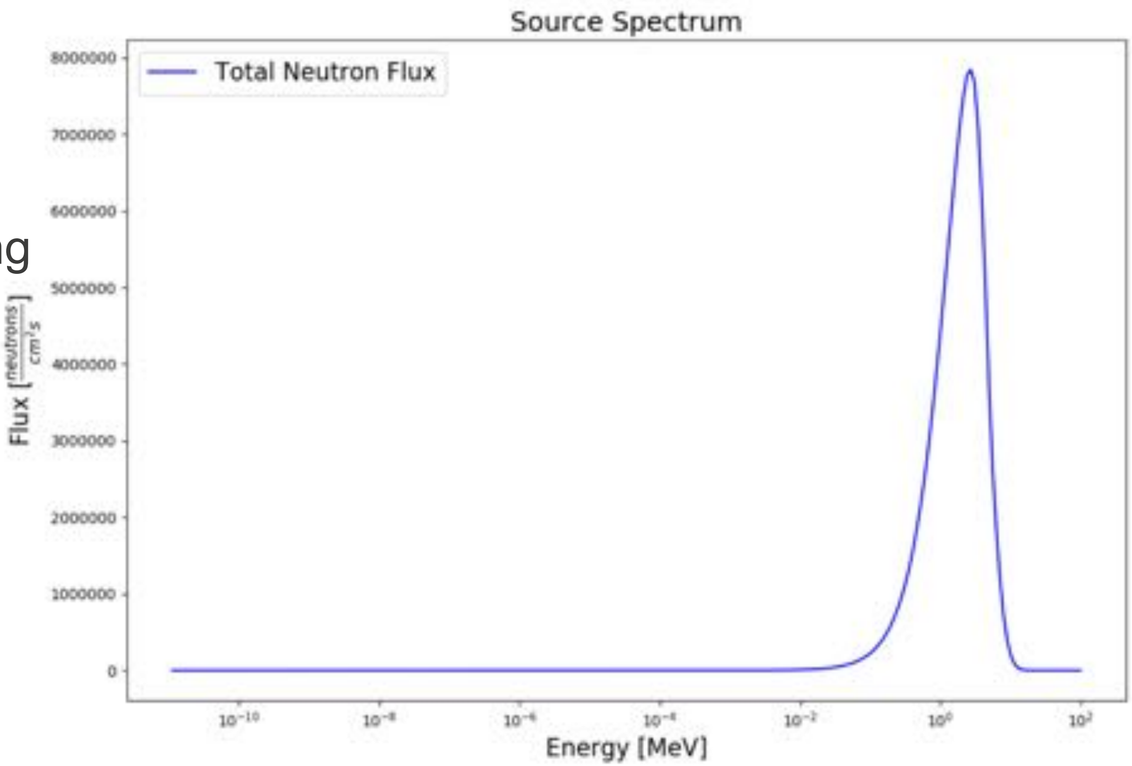
- Due to the complexity of the cask, $\frac{1}{8}$ of the cask was simulated





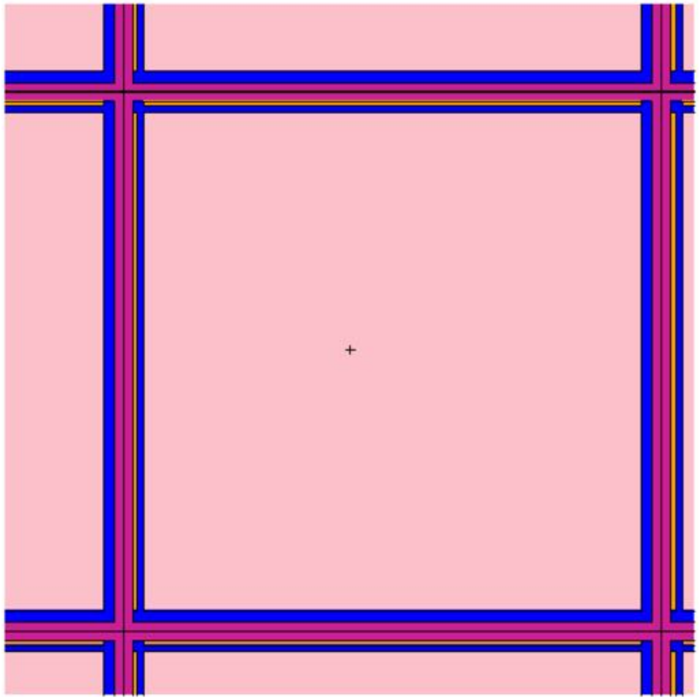
Current Work: Source term

- Source term for MCNP simulations and analytic models need to be found
- Next Generation Safeguards Initiative (NGSI) has a library of spent fuel compositions
  - Made for use with MCNP
  - Running MCNP in initialization mode creates a table detailing the composition of materials in the problem
- Compositions were used for ORIGEN-S models
  - 0-dimensional decay and irradiation code



Current Work: Sub-region Identification

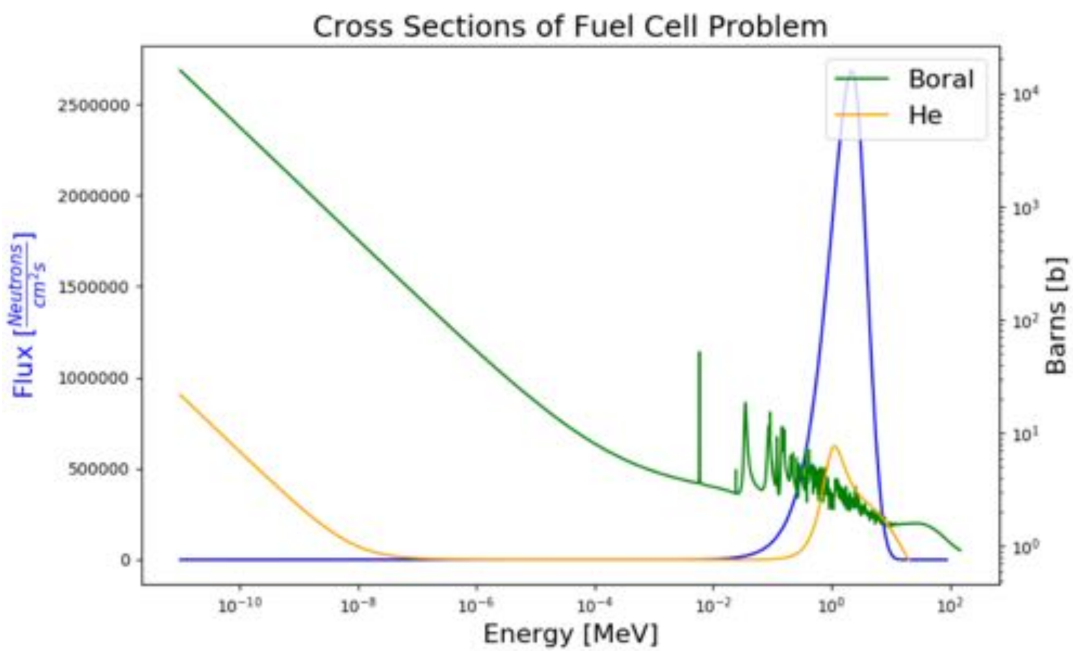
- A single fuel cell was chosen as the extent of the sub-region
  - Neutron flux through a Boral pad
    - Relatively monoenergetic flux
    - High thermal neutron absorption cross section



- Still need to determine how transport will be handled

Current Work: Analytic Model

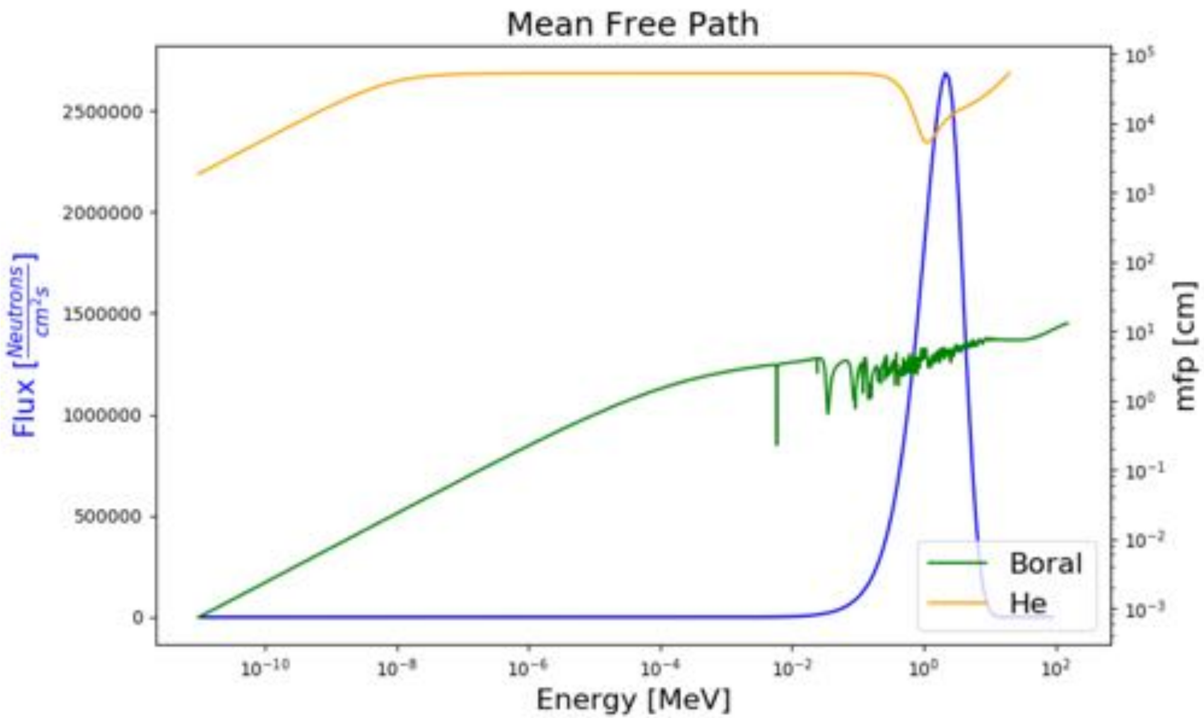
- Source spectrum is comprised mainly of fast neutrons
  - Maybe energy dependence of BTE can be handled in one or two groups?
- Comparing the cross sections of materials to determine how energy dependence is handled



- A two-group model allows for treatment of physics in fast and thermal regions separately

Current Work: Analytic Model

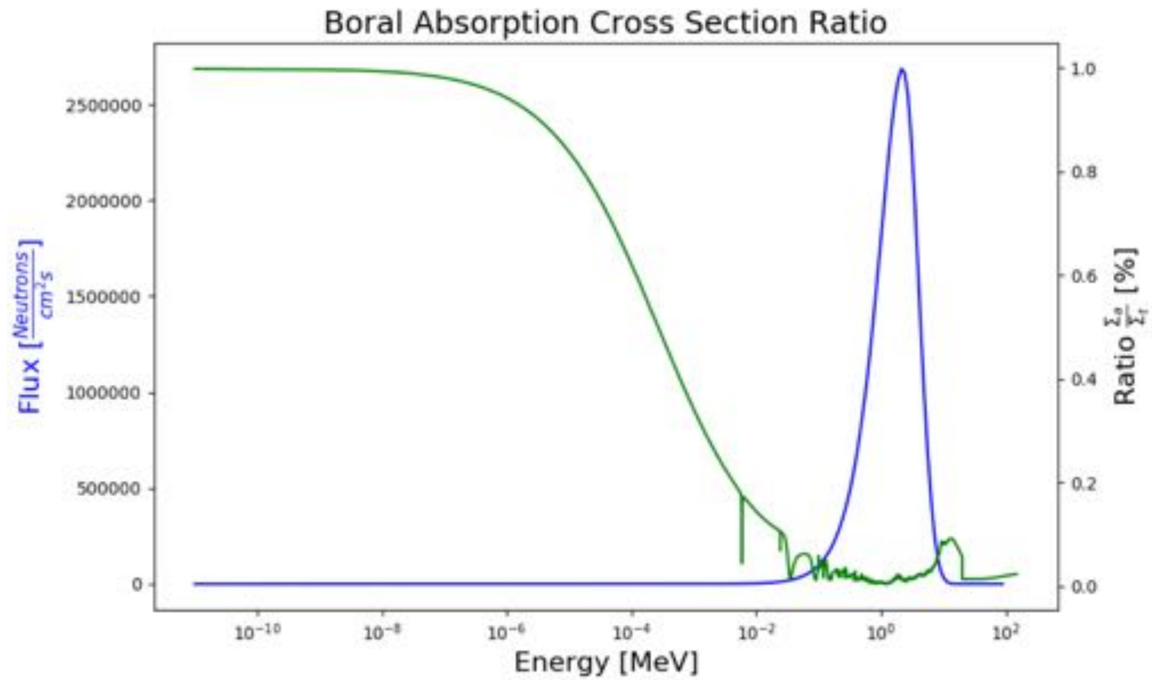
- The diffusion equation is a common representation of the BTE
  - There are known solutions



- The mean free path is on the order of (or higher than) the thickness of our Boral pad (~0.25 cm)
- BTE is a more appropriate choice of model

Current Work: Analytic Model

- The threshold between thermal and fast groups needs to be set



- Below 1eV, absorption processes comprise approximately 100% of interactions
- Setting the threshold at 1eV allows for further assumptions in each region to hold

Current Work: Analytic Model

- Two-group BTE
- Fast group (above 1eV)

$$\hat{\Omega}_x \frac{\partial \phi_1}{\partial x} + \Sigma_{t,1}\phi_1 = \Sigma_{s,11}\phi_1 + S_1$$

- Assumptions:
  - $S_1 = 0$
  - $\Sigma_{t,1} \approx \Sigma_{s,1}$
  - $\Sigma_{R,12} \equiv \Sigma_{s,1} - \Sigma_{s,11} = \Sigma_{s,12}$

$$\hat{\Omega}_x \frac{\partial \phi_1}{\partial x} + \Sigma_{R,12}\phi_1 = 0$$

- The only mechanisms present are due to scattering

- Thermal group (below 1eV)

$$\hat{\Omega}_x \frac{\partial \phi_2}{\partial x} + \Sigma_{t,2}\phi_2 = \Sigma_{s,12}\phi_1 + \Sigma_{s,22}\phi_2 + S_2$$

- Assumptions:
  - $S_2 = 0$
  - $\Sigma_{t,2} \approx \Sigma_{a,2}$
  - $\Sigma_{s,22} = 0$

$$\hat{\Omega}_x \frac{\partial \phi_2}{\partial x} + \Sigma_{a,2}\phi_2 = \Sigma_{R,12}\phi_1$$

- The only source of neutrons are those down-scattered from the fast group

Current Work: Cross Section Handling

- Group averaged cross sections were calculated
  - From Duderstadt and Hamilton

$$\langle \Sigma_x \rangle = \frac{\int_{\Sigma_{g-1}}^{\Sigma_g} \phi_g \Sigma_x}{\int_{\Sigma_{g-1}}^{\Sigma_g} \phi_g}$$

- The removal cross section governs the probability that a neutron will undergo a scattering event and be removed from group 1
- Lewis defines an approximation for the removal cross section

$$\Sigma_{R,12} \approx \frac{1}{n} \Sigma_s$$

- n is the number of collisions for a neutron to slow down from one energy to another
  - In Boral, $n \approx 125\ collisions$ for a neutron to slow from 1MeV to 1eV

Current Work: Analytic Solution

- The solutions to the fast and thermal group equations

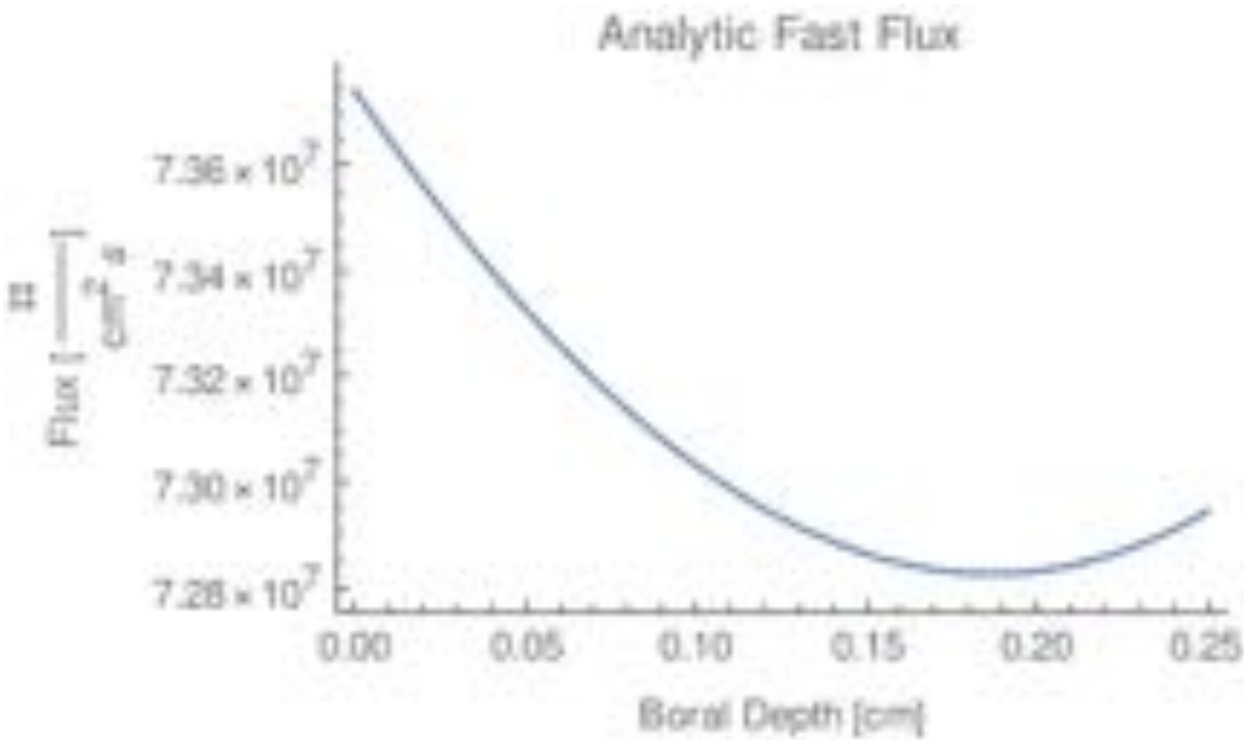
$$\phi_1(x) = \phi_f e^{-\frac{\Sigma_{R,12} x}{\mu}}$$

$$\phi_2(x) = \frac{\phi_f \Sigma_{R,12} e^{-\frac{\Sigma_{a,2} x}{\mu} + \frac{(\Sigma_{a,2} - \Sigma_{R,12}) x}{\mu}}}{\Sigma_{a,2} - \Sigma_{R,12}} + \frac{\phi_t(\Sigma_{a,2} - \Sigma_{R,12}) - \phi_f \Sigma_{R,12}}{\Sigma_{a,2} - \Sigma R, 12} (e^{-\frac{\Sigma_{a,2} x}{\mu}})$$

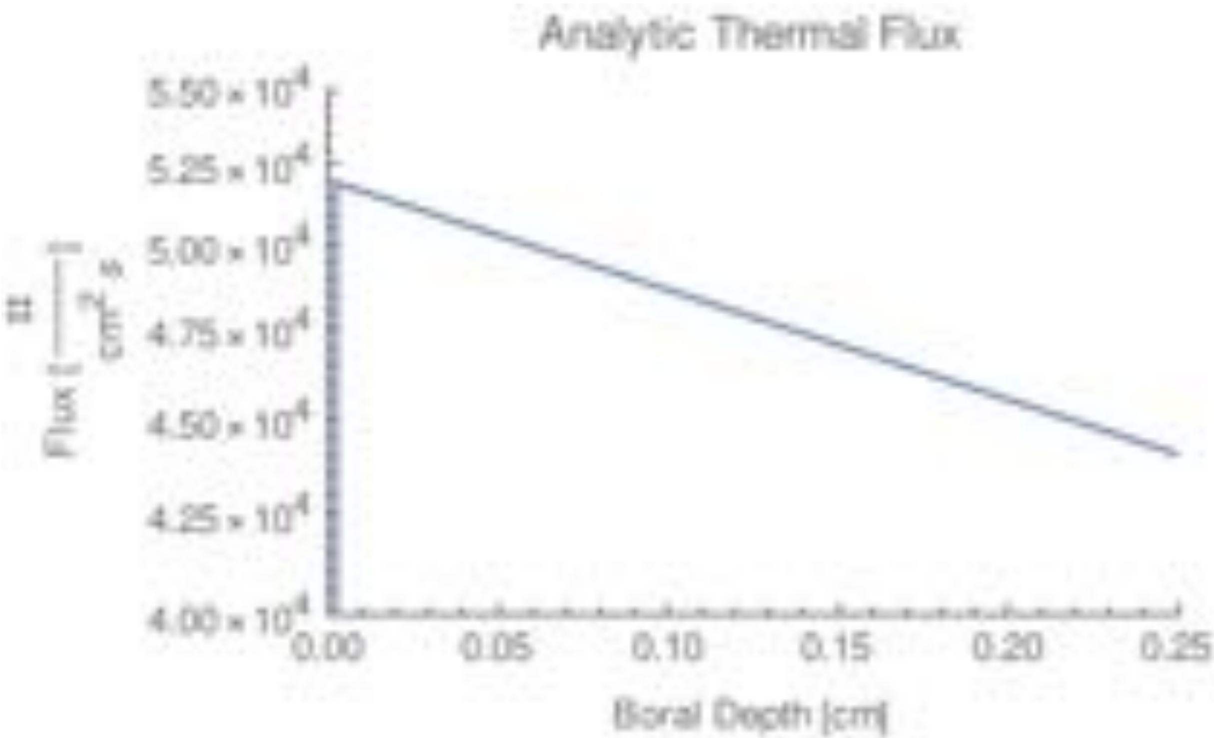
- ϕ_f is the portion of the source flux above 1eV
- ϕ_t is the portion of the source flux below 1eV

- A final correction was made to the analytic solutions
  - Since Boral pads are placed between two sources, a second source term was geometrically attenuated from the opposite side of the pad and added to each flux correspondingly

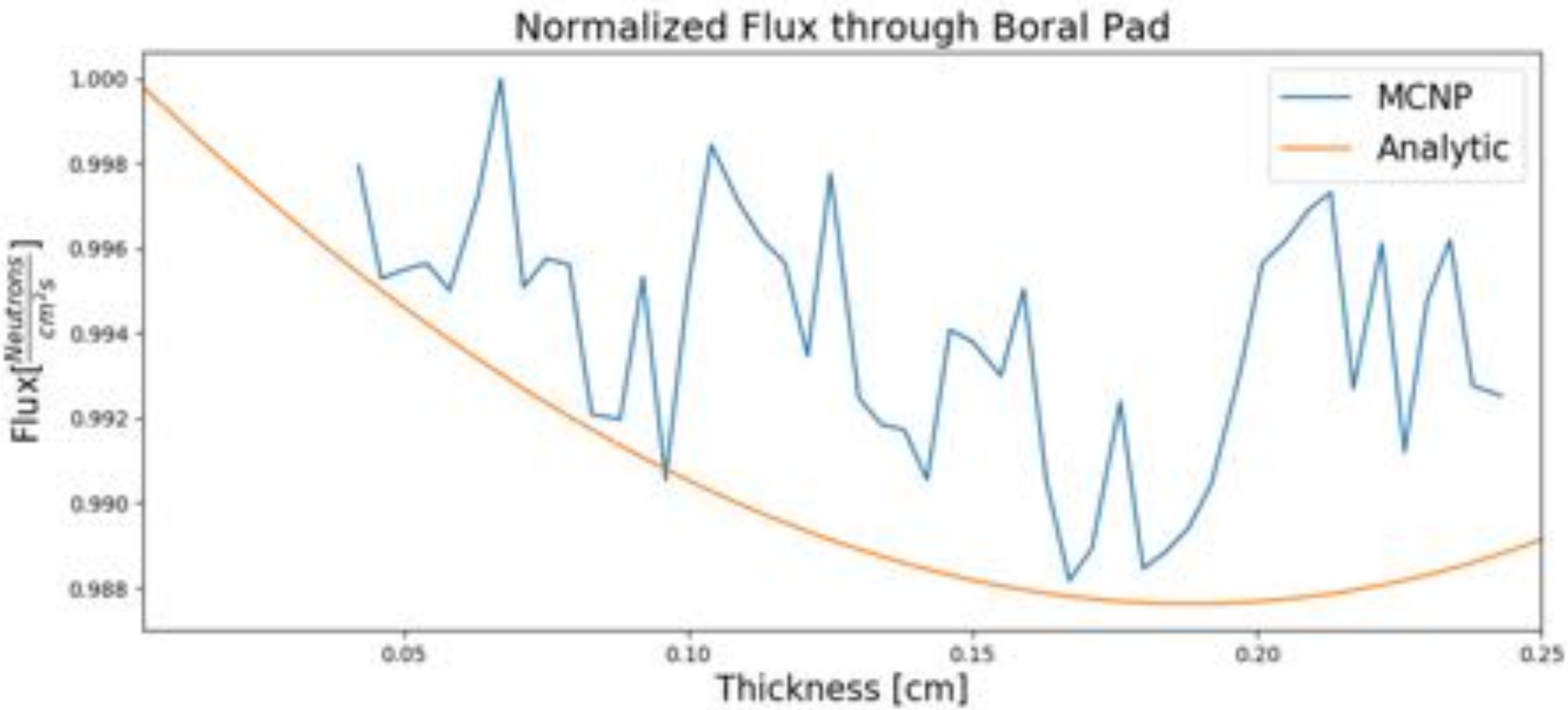
Current Work: Analytic Solution



Current Work: Analytic Solution



Current Work: Comparing Results



- The flux from MCNP shows a similar trend to the analytic solution

Current Work: Results

- What was learned
  - Input Correctness
    - Original mesh tally location extended 0.2 mm into stainless steel
    - Affect on flux was small
    - Original results appeared to be correct but did not match analytic results
    - Result was the tally location was fixed
  - Underlying physics
    - Flux is mainly fast
      - Scattering dominated
      - Slowing down processes is most prevalent
      - Thermal flux is strongly attenuated by Boral
        - » Hardens the flux
  - Future Investigations
    - Temperature affects on cross section
      - Doppler broadening of cross sections

Future Work

Future Work: Year 1

- Year 1 goals
  - More detailed MCNP geometry will be developed
    - Detailed fuel bundles
    - Change air vent structure
  - To show the versatility of the method, five to six more regions will be identified and analytic models will be compared with MCNP results
    - Flux through cement anulus
    - Flux through carbon steel shell
    - Flux through lid bottom plate
    - Flux through cement above MPC
    - Dose at cask surface (can be compared to literature)

Future Work: Year 2

- Year 2 goals
  - Identification of reoccurring parameters in analytic models for sensitivity analysis
    - Σ_t, Σ_a, Σ_s, Σ_R
  - MCNP sensitivity analysis of parameters
    - Vary cross section data through manual addition of uncertainty
    - S(α, β) cards to vary the cross section data based on temperature
  - FSAP sensitivity analysis
    - Common analytic equations from sub-regions
  - Comparison of sensitivity analysis results
  - Development of final methodology of analysis process
    - Focused on spent fuel cask modeling

Future Work: Papers and Presentations

- 3 expected papers
  - Symmetry analysis of simplified form of BTE
  - Verification of spent fuel cask simulations using analytic models
  - Sensitivity Analysis of neutron transport equations

- Presentations
  - American Nuclear Society
  - American Physical Society – Division of Nuclear Physics

Future Work: Timeline

Task	Fall '18	Spr '19	Sum '19	Fall '19	Spr '20	Sum '20	Fall '20
Identification of sub-regions	X						
Development of detailed MCNP geometry	X						
Apply theory to sub-regions		X	X				
Comparison of results			X				
ID of input parameters for Sensitivity Analysis				X	X		
Find sensitivities of BTE using FSAP					X	X	
Write dissertation						X	X

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Questions

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Backup Slides

Symmetry Analysis: Ensuring the Derivative is Preserved

$$z = \frac{dy}{dx}$$
$$z - \frac{dy}{dx} = 0$$
$$z * dx - dy = 0$$
$$pr\boldsymbol{X}(z * dx - dy) = 0$$
$$zpr\boldsymbol{X}(dx) + dxpr\boldsymbol{X}(z) - pr\boldsymbol{X}(dy) = 0$$

- Use $pr\boldsymbol{X}(dx) = d(pr\boldsymbol{X}(\boldsymbol{x}))$

$$\zeta + \left[\frac{\partial \eta}{\partial x} + \frac{\partial \eta}{\partial y} z\right] z - \left[\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} z\right] = 0$$
$$0 + [1 + 0z]z - [0 + z] = 0$$

Symmetry Analysis: Re-writing the derivative

$$\frac{\partial y}{\partial x} = \frac{\partial}{\partial x}(rx) = \frac{\partial r}{\partial x}x + r$$

Sensitivity Analysis: G-Derivatives

- Taking $\boldsymbol{e}^0 = (\boldsymbol{u}^0, \boldsymbol{\alpha}^0)$
- The most general and fundamental concept for the definition of the sensitivity of a response to variations in the system parameters is the G-derivative

$$\delta \boldsymbol{R}(\boldsymbol{e}^0; \boldsymbol{h}) \equiv \left\{ \frac{d}{dt} [\boldsymbol{R}(\boldsymbol{e}^0 + t\boldsymbol{h})] \right\}_{t=0} = \lim_{t \to 0} \frac{\boldsymbol{R}(\boldsymbol{e}^0 + t\boldsymbol{h}) - \boldsymbol{R}(\boldsymbol{e}^0)}{t}$$

- The G-differential of $\delta \boldsymbol{R}(\boldsymbol{e}^0; \boldsymbol{h})$ is related to the total variation $[\boldsymbol{R}(\boldsymbol{e}^0 + t\boldsymbol{h}) - \boldsymbol{R}(\boldsymbol{e}^0)]$of $\boldsymbol{R}$ at $\boldsymbol{e}^0$ through the relation

$$[\boldsymbol{R}(\boldsymbol{e}^0 + t\boldsymbol{h}) - \boldsymbol{R}(\boldsymbol{e}^0)] = \delta \boldsymbol{R}(\boldsymbol{e}^0; \boldsymbol{h}) + \Delta \boldsymbol{h}, with \ \lim_{t \to 0} \frac{[\Delta(t\boldsymbol{h})]}{t} = 0$$